If all quantities are evaluated at $\rho = a$, subtraction of (A.10) and (A.11) from (A.22) and (A.23) yields

$$H_{\phi n}(a+0) - H_{\phi n}(a-0) = q_{11n}(jE_{zn}) + q_{12n}E_{\phi n} \tag{A.27}$$

$$j[H_{zn}(a+0) - H_{zn}(a-0)] = q_{12n}(jE_{zn}) + q_{22n}E_{\phi n}. \tag{A.28}$$

Acknowledgment

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Wide-Band Equivalent Circuits of Microwave Planar Networks

GUGLIELMO D'INZEO, FRANCO GIANNINI, AND ROBERTO SORRENTINO, MEMBER IEEE

***Abstract*—A broad-band equivalent circuit of a generic microwave planar network is derived in terms of lumped constant elements. Contrary to previously proposed equivalent circuits, whose elements are strongly frequency dependent, the elements of the new one show only a smooth dependence on the frequency, because of the dispersion properties of microstrip structures. The equivalent circuit proposed is therefore easy to handle and is shown to be a useful basis for direct synthesis of planar structures. Good agreement with the theory is demonstrated by experiments performed on structures with different geometries up to 12.5 GHz, by using equivalent circuits whose elements are assumed to be constant with the frequency.**

I. Introduction

ONE OF THE major problems in the analysis and design of MIC's is that of determining the parasitics. In a general planar networks parasitics arise essentially from two distinct phenomena: the existence of fringing fields and the excitation of higher order modes at the discontinuities.

While an infinite microstrip line can be characterized by an effective width and an effective permittivity [1], such an approach is no more valid in the case of a planar circuit or, in particular, in the presence of discontinuities. In such cases, in fact, the electromagnetic (EM) field is no more a quasi-TEM one, but results from the contribution of more complicated field distributions to which a variation both of fringe effects and of the EM energy storage have to be ascribed.

A general method of analysis of microwave planar structures, which accounts for fringe field effects, has been recently presented [2]; the approach may be regarded as an extension of the magnetic wall model of microstrip lines [3], [4]. Fringe effects, in fact, are taken into account through effective dimensions and effective permittivities which depend on the field distribution inside the planar structure [5]. On the basis of this method, in the present paper the lumped element equivalent circuit, in which parasitics are automatically taken into account, is derived for a general planar network.

The equivalent circuit approach has been extensively

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applied to the particular case of a discontinuity between two lines of different characteristic impedances. In such a case the effect of fringing fields consists of an increased capacitance of the structure, while the magnetic energy stored in the higher order evanescent modes excited at the discontinuity can be characterized through a series inductor. Equivalent capacitors and inductors of a single step discontinuity between two semi-infinite microstrip lines have been calculated through a quasi-static approach [6], by Gopinath together with several coworkers [7]–[10]. The resulting equivalent circuit, however, cannot be applied at high frequencies, nor in the case of interacting discontinuities. In fact, if the frequency is not sufficiently low, the contribution of the evanescent higher order modes becomes important as to modify both the fringing field distribution and the EM energy storage at the discontinuity; in addition, when, as in practical cases, more than one discontinuity is present, the interaction between the discontinuities takes place through higher order modes. It follows that this approach is only valid well below cutoff of the first higher mode of the wider microstrip line; in particular, it cannot explain the existence of transmission zeros in planar structures [11], [12].

Following a different theoretical approach, Bianco *et al.* [11] have proposed broad-band equivalent circuits for single and double step discontinuities; fringe effects, however, are taken into account through a rather approximated technique, so that the applicability of these equivalent circuits is strongly limited.

The dynamic approach by Menzel and Wolff [12], which accounts for the frequency variation of the energy stored in the evanescent higher order modes, has been used by Kompa [13] to obtain a lumped-element equivalent circuit of an abrupt impedance step.

However, Kompa's equivalent circuit contains elements which are strongly variable with the frequency, so that it is not easy to handle when broad-band simulation is needed. On the contrary, the elements of the equivalent circuits proposed here are frequency dependent only because of the dispersion properties of microstrip circuits so that, in a first approximation, they may be assumed to be constant with the frequency, also in broad-band simulations. Finally, it should be stressed that, contrary to the equivalent circuits proposed early, the present one can be applied to structures having geometries different from the rectangular one.

II. The General Equivalent Circuit

A generic two-port microwave planar network is sketched in Fig. 1. Following [2], the analysis is carried out firstly by determining in the domain S of the xy plane the orthonormalized set of eigenfunctions of the bidimensional Helmholtz equation

$$\nabla^2\phi_m + k_m^2\phi_m = 0 \tag{1}$$

with homogeneous Neumann boundary conditions. In the hypotheses that losses are negligible, that higher order

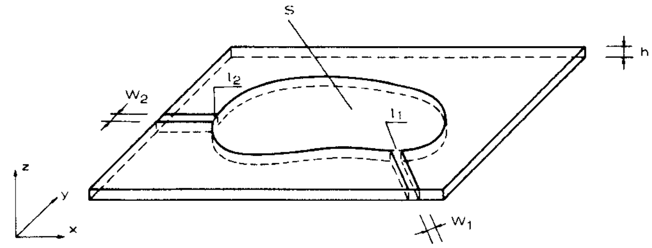

Fig. 1. The planar two-port network.

modes on the connecting lines are evanescent and that line widths are much smaller than the structure's dimensions, the impedance matrix of the network is given by

$$[Z] = \sum_{m=0}^{\infty} [Z_m] \tag{2}$$

with

$$[Z_m] = \frac{j\omega}{\omega_m^2 - \omega^2}\frac{h}{\epsilon_m}\begin{bmatrix} R_{m1}^2 & R_{m1}R_{m2} \\ R_{m2}R_{m1} & R_{m2}^2 \end{bmatrix}. \tag{2a}$$

In (2a) $\omega_m = k_m/\sqrt{\mu\epsilon_m}$ is the mth structure's resonant frequency

$$R_{mi} = \frac{1}{w_{i,\text{eff}}}\int_{l_i}\phi_m\,dl, \qquad i=1,2 \tag{2b}$$

is the coupling coefficient between the mth resonant mode and the TEM wave traveling on the ith line; l_i is the portion of the contour of the planar structure corresponding to the ith port; k_m^2 and ϕ_m are the mth eigenvalue and eigenfunction of (1), respectively; ϵ_m is the effective permittivity of the mth resonant mode [5]; h is the substrate's thickness; $w_{i,\text{eff}}$ is the effective width of the ith line [1].

The first term of the series (2), for $m=0$, has $\omega_0=0$ and thus corresponds to the mode resonating at zero frequency, for which $\phi_0 = 1/\sqrt{S}$, where S is the area of the planar network, and $R_{01}=R_{02}=1$. From (2)–(2b) it follows that, for a planar network of given geometry, its filtering properties depend on the coupling coefficients, i.e., on the positions and widths of the ports.

According to [5], fringe effects are taken into account in previous formulas by ascribing to the structure effective dimensions, and by introducing a different effective permittivity for each resonant mode. As a consequence, each resonant frequency of the planar structure is shifted with respect to the case of magnetic wall model. A typical example is the modal inversion between the TM_{410} and TM_{120} modes demonstrated in a circular microstrip [14].

As can be easily seen, each term (2a) of (2) can be realized in the form of an antiresonant LC cell (with the exception of the first term which corresponds to a pure capacitance) connected to two ideal transformers whose transformer ratios are given by the coupling coefficients (2b). This leads to the equivalent circuit shown in Fig. 2 as the series connection of an infinite number of such cells. This equivalent circuit is of the type of the one derived in [19]. However, there is a substantial difference between

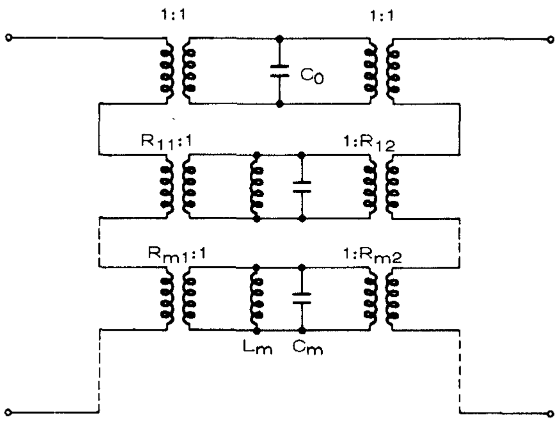

Fig. 2. The general equivalent circuit of a two-port planar network.

them, since each LC cell of the equivalent circuit of Fig. 2 is derived from an effective model (dimensions and permittivity) of the planar structure, which is in general different for each resonant mode.

For practical application, only a finite number of such resonant cells has to be taken into account, depending on the frequency range of interest and on the approximation to be obtained. It is worth noting that the broad-band equivalent circuits previously proposed [11], [13] contain elements which are so strongly frequency dependent as to become infinite at certain frequencies. On the contrary, the equivalent circuit elements of Fig. 2 are only smoothly frequency dependent because of the well-known dispersion characteristic of the microstrip structure and therefore have always finite values. Furthermore, in most cases, the frequency dependence can be neglected with a fairly good approximation. In all the experiments reported below, in fact, a good agreement is shown up to 12.5 GHz, through the use of equivalent circuits whose elements are assumed to be frequency independent.

Typical features of a microwave planar network can be shown through the equivalent circuit of Fig. 2. For example, suppose the signal frequency coincides with the resonant frequency ω_n of the nth cell; it is immediately found that the transformer radii of such a cell, i.e., the coupling coefficients of the corresponding mode, determine the relationship between the currents I_1, I_2 at the ports of the circuit

$$R_{n1}I_1 + R_{n2}I_2 = 0.$$

If $R_{n1} \neq 0$ and $R_{n2} = 0$, i.e., the nth mode is uncoupled to port 2, then I_1 must be zero, and the circuit presents an open circuit at the port 1 for $\omega = \omega_n$. This has been called a modal transmission zero of the microwave planar network [2], [15]. On the contrary suppose $|R_{n1}| = |R_{n2}|$; if, for $\omega = \omega_n$, the contribution of all the other cells is negligible, as it generally happens, one obtains

$$V_1/I_1 = V_2/(-I_2).$$

This means that the impedance at the second port is transferred at the first port, thus $\omega = \omega_n$ is a frequency of null attenuation.

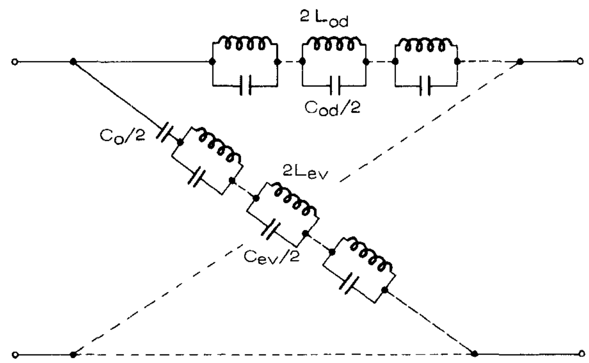

Fig. 3. The equivalent circuit of a two-port symmetrical structure.

Therefore, depending on the coupling coefficients, the same resonant mode of the microwave structure can give place either to a transmission zero or to a reflection zero.

III. Symmetrical Structures

The equivalent circuit of Fig. 2 is generally of impractical use. A more suitable equivalent circuit can be derived in the case of symmetrical structures. When the planar network is symmetrical, the modal solutions of (1) can be divided into even and odd modes, depending whether ϕ_m has equal or opposite values at the two ports. As a consequence, the coupling coefficients are related by

$$R_{m2} = \alpha_m R_{m1} \tag{3}$$

where $\alpha_m = 1$ for even modes, $\alpha_m = -1$ for odd modes. Separating the contributions of the even and odd modes, the impedance parameters (2) of a symmetrical network can be written, because of (3)

$$Z_{11} = Z_{22} = \Sigma_{\text{ev}} \frac{j\omega h R_{\text{ev}}^2/\epsilon_{\text{ev}}}{\omega_{\text{ev}}^2 - \omega^2} + \Sigma_{\text{od}} \frac{j\omega h R_{\text{od}}^2/\epsilon_{\text{od}}}{\omega_{\text{od}}^2 - \omega^2}$$
$$= \frac{1}{2}(Z_{\text{ev}} + Z_{\text{od}})$$
$$Z_{12} = Z_{21} = \Sigma_{\text{ev}} \frac{j\omega h R_{\text{ev}}^2/\epsilon_{\text{ev}}}{\omega_{\text{ev}}^2 - \omega^2} - \Sigma_{\text{od}} \frac{j\omega h R_{\text{od}}^2/\epsilon_{\text{od}}}{\omega_{\text{od}}^2 - \omega^2}$$
$$= \frac{1}{2}(Z_{\text{ev}} - Z_{\text{od}}) \tag{4}$$

where ev and od stand for even and odd, respectively. Expressions (4) lead to the symmetrical lattice realization of Fig. 3, which appears particularly useful since it avoids use of transformers. As is known, such a two-port network presents a pole of attenuation when the signal frequency is such that

$$Z_{\text{ev}}(\omega) = Z_{\text{od}}(\omega). \tag{5}$$

Now, since Foster's theorem applies to both Z_{ev} and Z_{od}, which are pure reactances, it is evident that (5) is satisfied if Z_{ev} remains finite between two consecutive poles of Z_{od}, or vice versa. In other words, if two consecutive resonant frequencies of the structure are both even (odd), then a pole of attenuation takes place between them. This has been called an interaction transmission zero [2], [15]. On

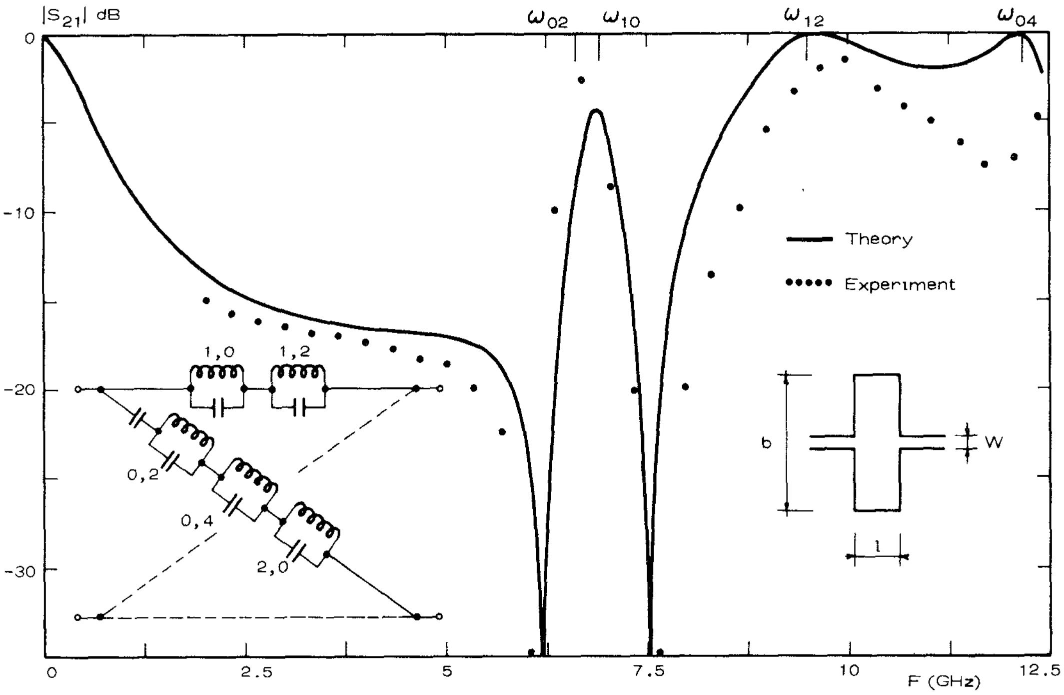

Fig. 4. Comparison between the experimental $|s_{21}|$ of a rectangular structure and the theoretical $|s_{21}|$ of the equivalent circuit indicated. (Alumina substrate, $\epsilon_r = 10$, $h = 0.0635$ cm; $l = 0.68$ cm, $b = 1.5$ cm, connecting line impedance $\eta = 50\ \Omega$).

the other hand, in a symmetrical lattice structure, a frequency of null attenuation occurs when

$$Z_{ev}(\omega)Z_{od}(\omega) = \eta^2 \tag{6}$$

where η is the characteristic impedance of the connecting lines. In the limit when η tends to infinite, condition (6) is satisfied when ω is coincident with a pole of Z_{ev} or Z_{od}, i.e., with a resonant frequency of the structure. As can be easily inferred from (2)–(2b), in fact, for $\eta \to \infty$, the impedance parameters of the planar network tend to finite values, except at the resonant frequencies. In practical cases, because of the finite widths of the lines, the frequencies of null attenuation are shifted from the resonant frequencies [16]. In some cases, however, it happens that a resonant mode does not give place to a reflection zero except for η greater than a minimum value. In fact, let us rewrite condition (6) in the form

$$X_{ev} = -\eta^2 / X_{od} \tag{6a}$$

where $X_{ev} = -jZ_{ev}$, $X_{od} = -jZ_{od}$. Since, because of Foster's theorem, X_{ev} is an always increasing function of the frequency, in a suitable interval enclosing a pole ω_{ev}, X_{ev} assumes all the values from $-\infty$ to $+\infty$; if $1/X_{od}$ remains finite in this interval, then (6a) is necessarily satisfied in that interval at a frequency $\bar{\omega}$; the higher η the closer $\bar{\omega}$ to ω_{ev}. On the contrary, if $1/X_{od}$ has a pole near ω_{ev}, it can be easily inferred that (6a) is satisfied only for sufficiently high values of η.

An example of application of the equivalent circuit of Fig. 3 for characterizing the frequency behavior of a rectangular planar structure in shown in Fig. 4, where the scattering parameter $|s_{21}|$ of the equivalent circuit is shown as a function of frequency in the range 0–12.5 GHz and is compared with the experimental measurements. As can be seen, good agreement is obtained by taking into account only the first six resonant modes of the structure, five of which fall in the range 0–12.5 GHz. Two poles of attenuation are located between the resonant frequencies $\omega_{00} - \omega_{02}$ (even modes) and $\omega_{10} - \omega_{12}$ (odd modes), according to the rule previously stated. Because of the presence of these poles of attenuation, produced by the interaction between resonant modes involving both an x- and y-field dependence, this structure may not be analyzed through a quasi-static approach, except well below 5 GHz. Let us observe, moreover, that while each one of the resonant frequencies ω_{00}, ω_{12}, ω_{04}, gives place to a reflection zero, ω_{02} and ω_{10} do not. This is due to the fact that the connecting lines impedances are too low (50 Ω) so that condition (6) is not satisfied both for ω_{02} and ω_{10} (see Fig. 5(a)). On the contrary, condition (6) would be satisfied if the connecting lines would have a very high impedance (see Fig. 5(b)).

So far, we have implicitly assumed that only one independent eigensolution of (1) exists for each k_m^2. On the contrary, in typical cases such those of circular, annular or square structures, (1) has two linearly independent solutions corresponding to the same eigenvalue, so that there are two resonant modes with the same resonant frequency. If the two degenerate modes are one even and the other one odd, both the even and the odd branches of the equivalent lattice circuit (Fig. 3) become open circuits at the same resonant frequency, giving place to a transmission zero. This type of zero has been classified among the modal zeros [2] since it occurs at a structure's resonant

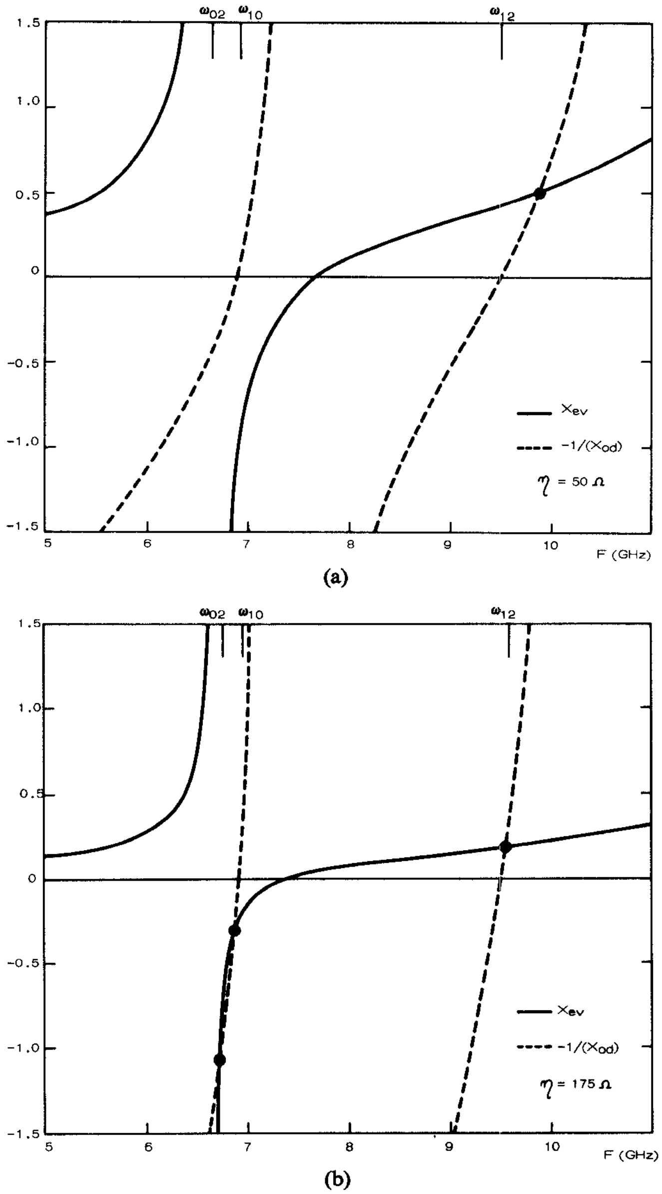

Fig. 5. Plots of X_{ev} and $-1/X_{od}$ of the rectangular structure of Fig. 4 with (a) $\eta = 50\ \Omega$, and (b) $\eta = 175\ \Omega$.

frequency. Unless the circuit of Fig. 3 is valid also in case of degenerate structures, some simplifications can be made accounting for the mode degeneracy.

As an example, let us consider an annular two-port planar network. According to the analysis of this microwave structure [17], which is based on [2], the elements of the even and odd branches of the equivalent lattice structure can be written as follows:

$$C_{mn}^{(ev)} = \frac{\epsilon_{mn}}{hR_{mn}^2(1+\cos m\psi)};\ C_{mn}^{(od)} = \frac{\epsilon_{mn}}{hR_{mn}^2(1-\cos m\psi)}$$

$$L_{mn}^{(ev)} = \frac{\mu hR_{mn}^2(1+\cos m\psi)}{k_{mn}^2};\ L_{mn}^{(od)} = \frac{\mu hR_{mn}^2(1-\cos m\psi)}{k_{mn}^2} \tag{7}$$

where ψ is the angle between the connecting lines, ϵ_{mn} is the effective permittivity of the degenerate (m, n) mode

$$R_{mn} = 1/\sqrt{\pi}\, A_{mn} F_{mn}(k_{mn} r_{out}) \sin m\varphi / m\varphi$$

$$F_{mn}(k_{mn} r) = J_m(k_{mn} r) + K N_m(k_{mn} r)$$

$$A_{mn} = \delta_m \left\{ \int_{r_{in}}^{r_{out}} r [F_{mn}(k_{mn} r)]^2 dr \right\}^{-1}$$

$$K = -J_m'(k_{mn} r_{out}) / N_m'(k_{mn} r_{out})$$

$$\delta_{in} = \begin{cases} 1, & m = 0 \\ 2, & m \neq 0 \end{cases}.$$

φ is the angle subtended by the ports, and r_{in}, r_{out} are the inner and outer radii of the ring, respectively.

Expressions (7) show that, except when $|\cos m\psi| = 1$, each resonant frequency corresponds to a modal zero,

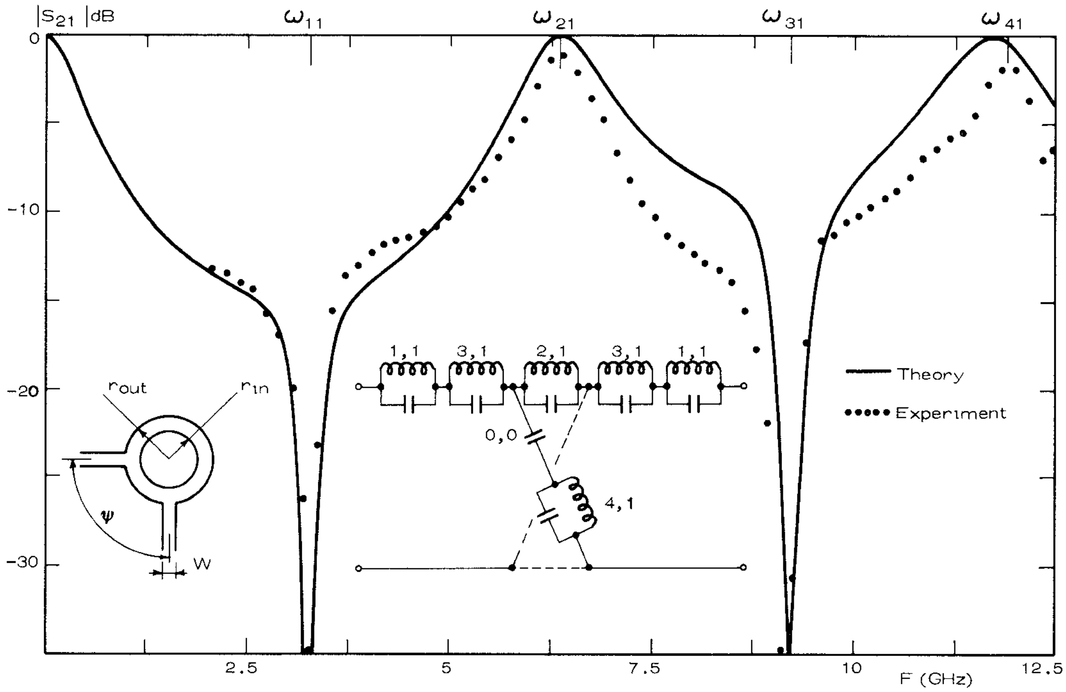

Fig. 6. Comparison between the experimental $|s_{21}|$ of an annular structure and the theoretical $|s_{21}|$ of the equivalent circuit indicated. (Epsilam substrate, $\epsilon_r = 10$, $h = 0.0635$ cm; $r_{in} = 0.4$ cm, $r_{out} = 0.7$ cm, $w = 0.06$ cm, $\psi = 90°$).

TABLE I

Mode	C(pF)	L(nH)	f res(GHz)
0,0	8.61555	–	0
1,1	3.96161	5.99266	3.26641
2,1	3.27220	1.91647	6.35543
3,1	2.71207	1.10168	9.20741
4,1	2.37537	0.75450	11.88834

since the corresponding degenerate modes contribute to both the even and odd parts of the $[Z]$ matrix. In particular, if $\cos m\psi = 0$, the lattice structure can be modified as to realize the corresponding pole of the Z matrix as a private pole of $Z_{11} = Z_{22}$. On the contrary, if $\cos m\psi = 1$, or $\cos m\psi = -1$, a resonant cell will appear only at the even, or odd, branch of the lattice equivalent circuit so that, according to the previous discussion, it generally gives place to a reflection zero. Fig. 6 shows the experimental and theoretical behaviors of the scattering parameter $|s_{21}|$ of an annular microstrip with orthogonal connecting lines ($\psi = \pi/2$) versus the frequency. The equivalent circuit used for the theoretical simulation is indicated in the same figure, while the values of the LC elements are quoted in Table I. The resonant frequencies ω_{11} and ω_{31}, for which $\cos m\psi = 0$, give place to two transmission zeros corresponding to two private poles of $Z_{11} = Z_{22}$ (see equivalent circuit) while frequencies of null attenuation are located close to the resonant frequencies ω_{21}, ω_{41} (for which $|\cos m\psi| = 1$).

The availability of lumped-element equivalent circuits of microwave planar network, besides allowing a simple characterization of their behavior, may be also the basis for obtaining synthesis procedures of these structures, starting from the conventional synthesis procedure of lumped element circuits.

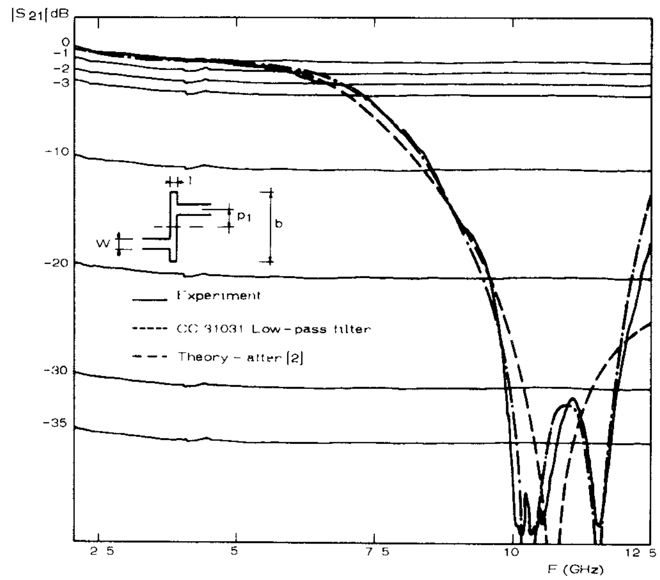

Fig. 7. $|s_{21}|$ of the filtering structure synthesized versus the frequency. (Alumina substrate, $\epsilon_r = 10$, $h = 0.0635$ cm, $l = 0.020$ cm, $b = 0.845$ cm, $w = 0.060$ cm, $p_1 = 0.162$ cm, reference frequency $f_r = 5$ GHz).

An example is illustrated in Fig. 7 where the measured scattering parameter $|s_{21}|$ of a rectangular two-port structure, realizing a Cauer–Chebycheff low-pass filter is plotted versus the frequency.

The rectangular structure has been synthesized according to [18]. For comparison, the same figure shows the behaviors of the CC 31031 prototype and that obtained according to [2].

IV. Conclusions

A broad-band equivalent circuit of a generic microwave planar network has been derived in terms of lumped-constant elements. These elements are only smoothly frequency dependent, because of the dispersion properties of microstrips, so that they may be considered, with good approximation, to be constant with the frequency, even in broad-band simulations.

Contrary to previously proposed equivalent circuits, which are strongly frequency dependent, the present one is easy to handle and can be a useful basis for designing microstrip planar structures starting from conventional synthesis procedures.

Experiments performed up to 12.5 GHz on structures with different geometries have shown good agreement with theoretical results.

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High-Accuracy Numerical Data on Propagation Characteristics of α-Power Graded-Core Fibers

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***Abstract*—High-accuracy data of normalized cutoff frequencies, propagation constants, and delay time of LP_{ml} modes for α-power graded-core fibers ($\alpha = 1$, 2, 4, and 10) are obtained by using two entirely different methods: power-series expansion and finite element methods, and the results are compared. The difference between cutoff frequencies obtained by these methods is less than 0.005 percent for most of the LP modes. The obtained data are accurate enough to be used as the standard for estimating the accuracy of other various analyses.**

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I. Introduction

VARIOUS methods have been presented for the analysis of propagation characteristics of optical fibers having arbitrary refractive-index profiles. Examples of those are the WKB method, [1] power-series expansion method, [2] Rayleigh–Ritz method [3], finite element

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